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Seminar 51 – Energy Efficiency of Novel and Conventional Compressors using Low-GWP Refrigerants

Thermodynamic Models for Predicting Compressor Performance with Low-GWP Refrigerants

CHICAGO 2015 Winter Conference

Learning Objectives

- Describe the main elements of a thermodynamic compressor model
- Explain how compressor models can be used to improve overall system performance with both traditional and alternative refrigerants
- Define the key operating and thermophysical properties that determine the mass flow rate and power consumption of a positive displacement compressor
- Describe the method of Kriging
- Explain the compression mechanisms of four different types of novel compressors
- Describe the difference between technical viability and commercial success for four different types of novel compressors

Acknowledgements

- **Craig Bradshaw** –
  Manager of Research and Development at Torad Engineering

- **Ian Bell** –
  Postdoctoral Researcher at University of Liege

Outline

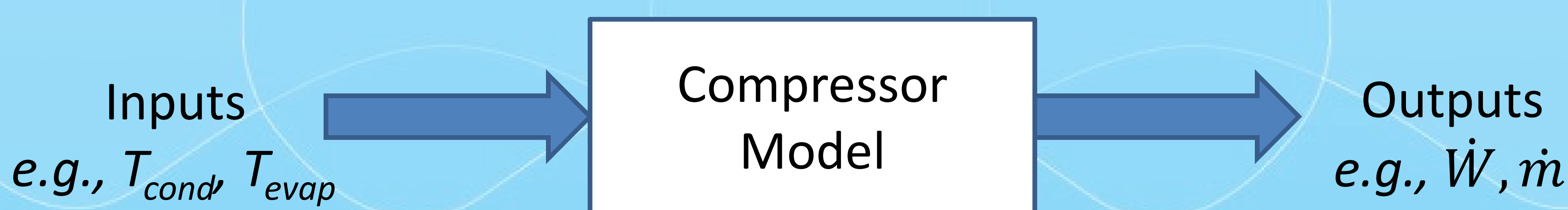
- **Motivation**
- **Types of compressor models**
- Components of a thermodynamic model
- Incorporating alternative refrigerants
- Case study
  - R-410A, R-32, NH_3, and CO_2
- Conclusions

Motivation

- Many refrigerants have high global warming potential (GWP)
- Work continues to develop low-GWP alternatives
  - 2,088 for R-410A
  - 543 for R-32
  - 1 for CO_2
  - 0 for NH_3
- Models provide cost- and time-effective method for predicting impact of alternative refrigerants

Compressor Models:
Introduction

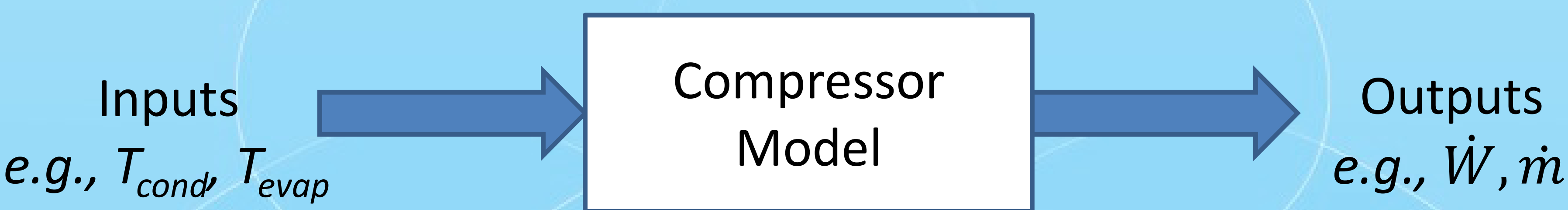
- Compressor models can be used to predict:
  - Power consumption, $\dot{W}$
  - Mass flow rate, $\dot{m}$
  - Volumetric or isentropic efficiency, η_V or η_s
- Model inputs may include:
  - Condensing temperature or pressure, T_{cond} or p_{cond}
  - Evaporating temperature or pressure, T_{evap} or p_{evap}
  - Superheat or subcooling, ΔT_{SH} or ΔT_{sub}



Compressor Models:
Types

Empirical or Statistical Models

- Developed based on experimental data
- Advantages:
  - Simplest calculation method
- Disadvantages:
  - Require data to develop
  - Restricted to a specific refrigerant and compressor



Compressor Models:
Types

Empirical or Statistical Models

- ANSI/AHRI Standard 540

$$X = C1 + C2 \cdot (S) + C3 \cdot D + C4 \cdot (S^2) + C5 \cdot (S \cdot D) + C6 \cdot (D^2) + C7 \cdot (S^3) + C8 \cdot (D \cdot S^2) + C9 \cdot (S \cdot D^2) + C10 \cdot (D^3)$$

where
C = Equation coefficient, represents compressor performance
S = Suction dew point temperature, °F
D = Discharge dew point temperature, °F

X = Any of the following variables:
- Power Input, W or kW
- Mass flow rate, lb/h
- Current, A
- Compressor Efficiency

Compressor Models:
Types

Thermodynamic Models

- Developed based on compressor geometry and thermodynamic principles assuming 1-D control volumes
- Advantages:
  – Physically based
  – Can predict impact of changes in geometry or refrigerant
- Disadvantages:
  – Detailed models require time investment to develop
  – Can be computationally expensive
  – Do not predict spatial variations in properties within CVs

Compressor Models:
Types

Computational Fluid Dynamic (CFD) Models

- Developed based on compressor geometry and thermodynamic principles assuming 3-D control volumes
- Advantages:
  – Physically based
  – Can predict impact of changes in geometry or refrigerant
  – Can provide information about 3-dimensional property variations
- Disadvantages:
  – Detailed models require time investment and detailed compressor information to develop
  – Computationally expensive

Outline

- Motivation
- Types of compressor models
- **Components of a thermodynamic model**
- Incorporating alternative refrigerants
- Case study
  - R-410A, R-32, NH_3, and CO_2
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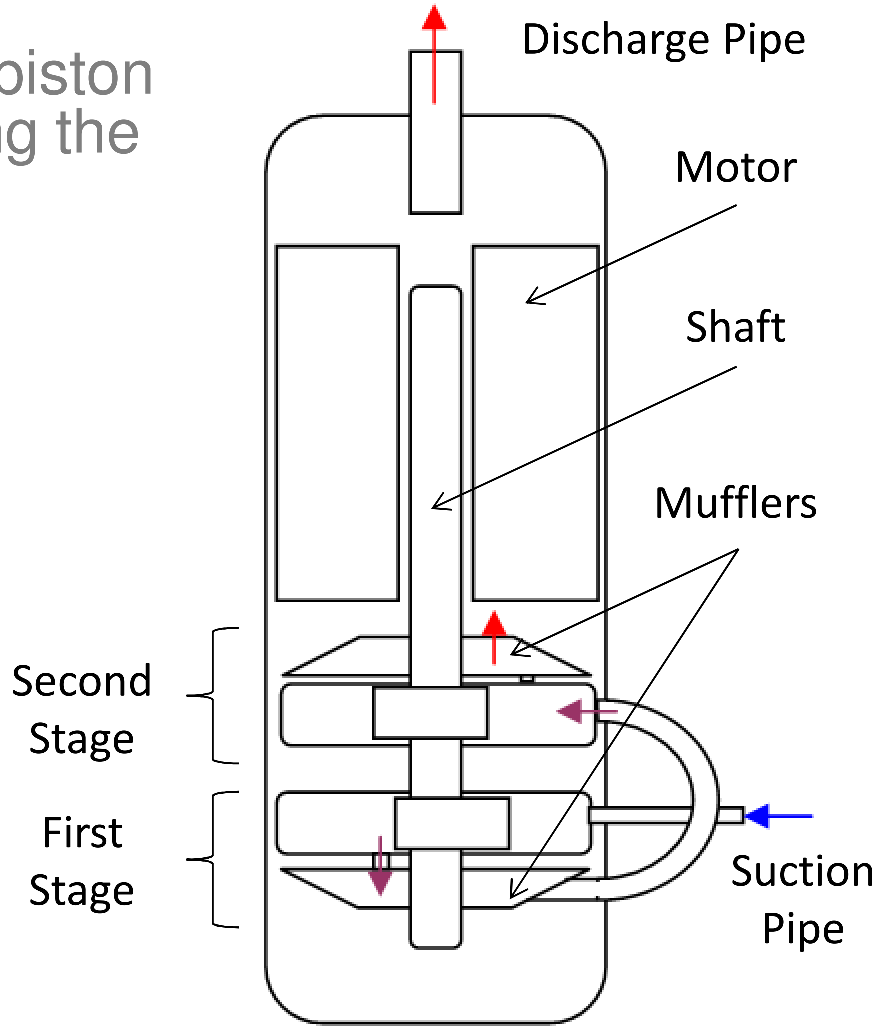
Thermodynamic Models:
Outline of Approach

1. Divide compressor into control volumes (CVs)
2. Apply mass and energy balance to each CV assuming
   - Spatially uniform properties within the CV
   - Compressor operates at steady state
3. Solve for temporal variations in properties
4. Integrate for time-averaged performance (e.g., power consumption, mass flow rate and efficiency)

Thermodynamic Models:
Identify CVs

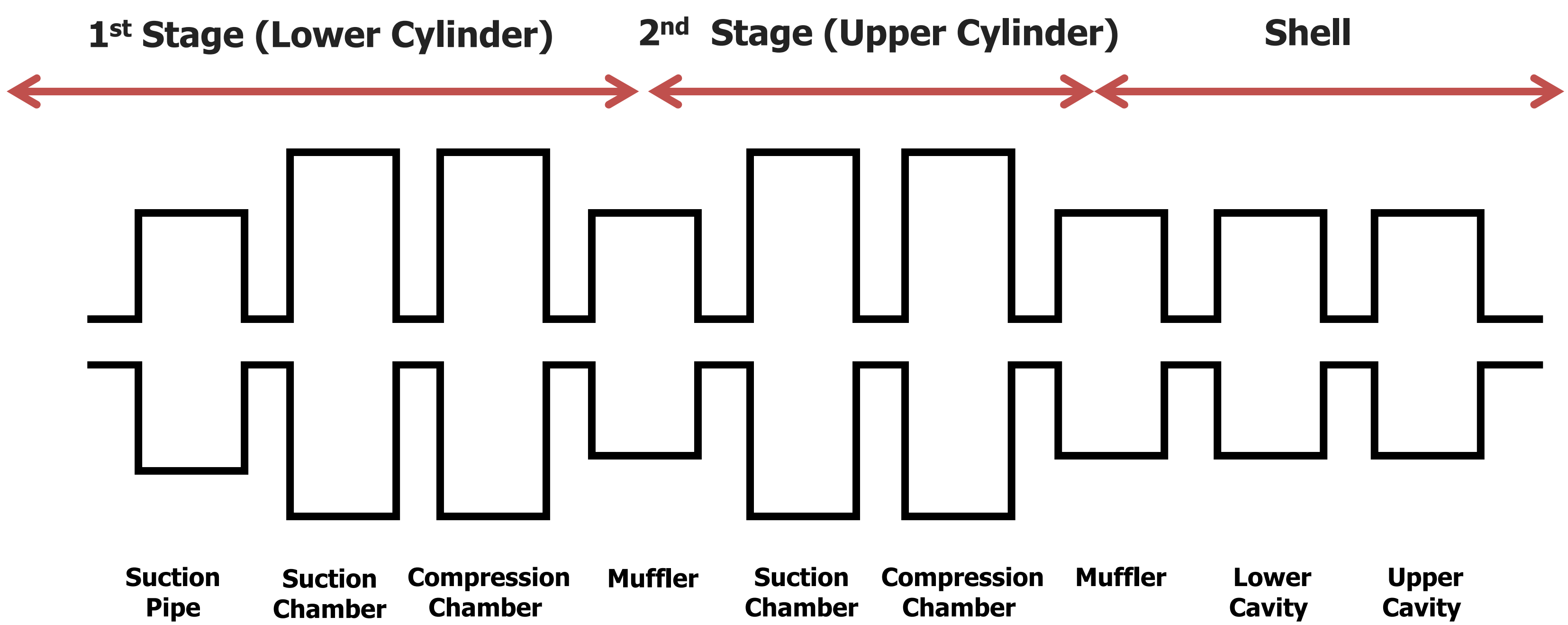
For example, a two-stage rolling piston compressor can be modeled using the following CVs:

- First stage
  - Suction chamber
  - Compression chamber
  - Muffler
- Second Stage
  - Suction chamber
  - Compression chamber
  - Muffler
- High pressure shell
  - Lower cavity
  - Upper cavity



Thermodynamic Models:
Identify CVs

For example, a two-stage rolling piston compressor may be modeled using the following CVs:



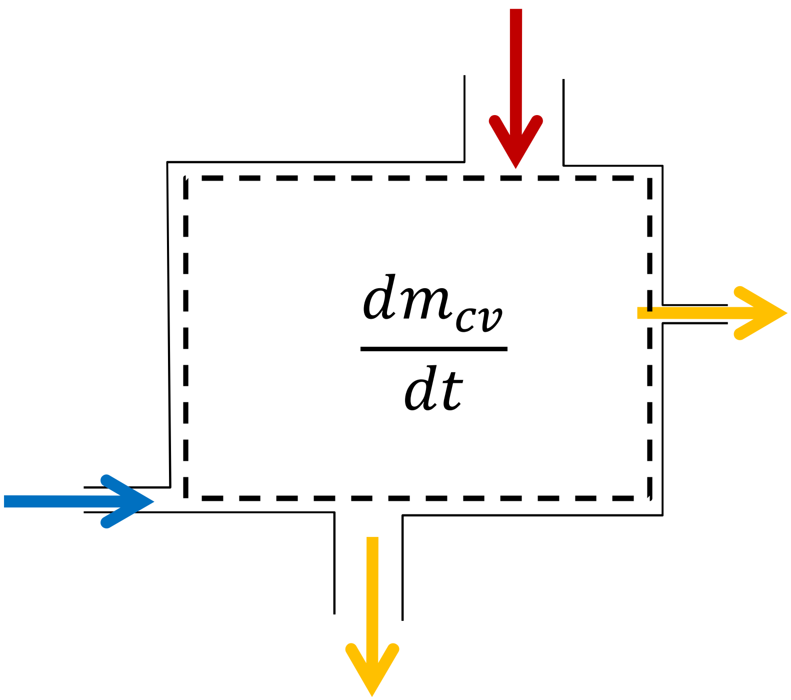
Thermodynamic Models:
Outline of Approach

1. Divide compressor into control volumes (CVs)
2. **Apply mass and energy balance to each CV** assuming
   - Spatially uniform properties within the CV
   - Compressor operates at steady state
3. Solve for temporal variations in properties
4. Integrate for time-averaged performance (e.g., power consumption, mass flow rate and efficiency)

Thermodynamic Models:
Conservation of Mass

- General form of mass balance:

$$\frac{dm_{cv}}{dt} = \sum \dot{m}_{in} - \sum \dot{m}_{out}$$



Assume:
- Quasi-equilibrium process (uniform properties within CV)
- Instantaneous mixing

Thermodynamic Models:
Conservation of Mass

- General form of mass balance:

$$\frac{dm_{cv}}{dt} = \frac{d(\rho V)}{dt} = \sum \dot{m}_{in} - \sum \dot{m}_{out}$$

- Expanded form:

$$V\frac{d\rho}{d\theta} + \rho\frac{dV}{d\theta} = \left(\sum \dot{m}_{in} - \sum \dot{m}_{out}\right)\frac{1}{\omega}$$

(properties)

(geometry)

(mass flow)

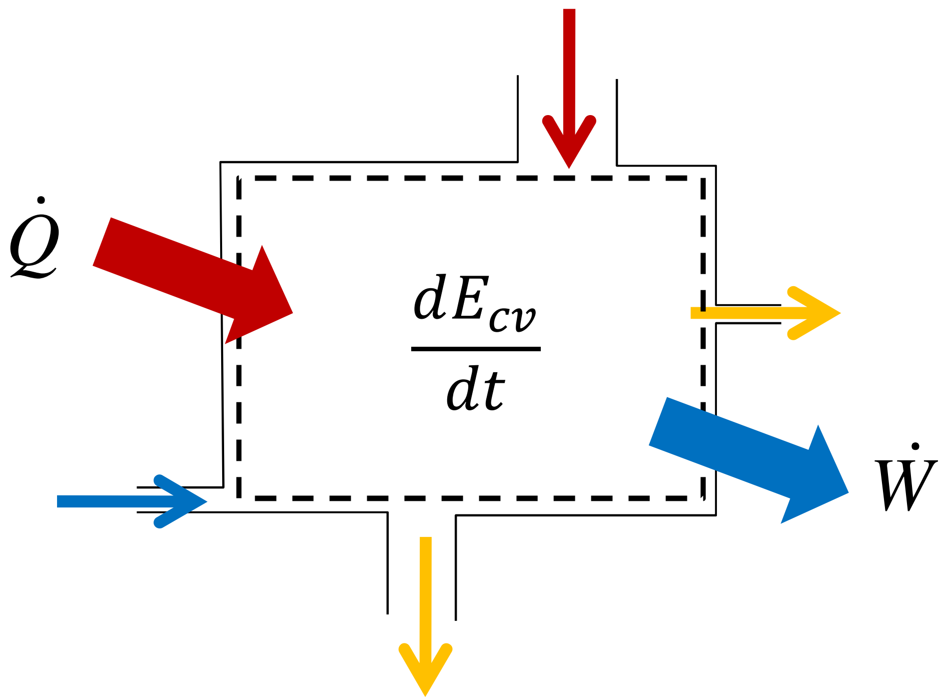
$\dot{m}_{in}$ = Mass flow rate in, kg/s
$\dot{m}_{out}$ = Mass flow rate out, kg/s
t = Time, s
V = Volume of chamber, m^3

θ = Crankshaft angle, degrees
ρ = Density, kg/m^3
ω = Rotational speed of crank, deg/s

Thermodynamic Models:
Conservation of Energy

- General form of energy balance:

$$\frac{dE_{cv}}{dt} = \sum \dot{m}_{in} h_{in} - \sum \dot{m}_{out} h_{out} + \dot{Q} - \dot{W}$$



Assume:
- Quasi-equilibrium process (uniform properties within CV)
- Instantaneous mixing
- Negligible changes in kinetic and potential energy

Thermodynamic Models:
Conservation of Energy

- General form of energy balance:

$$\frac{dE_{cv}}{dt} = \sum \dot{m}_{in} h_{in} - \sum \dot{m}_{out} h_{out} + \dot{Q} - \dot{W}$$

(properties) (mass flow)
(geometry) (heat transfer)

- Expanded form:

$$\left(\rho V \frac{\partial u}{\partial T}\right)\frac{dT}{d\theta} + \left(\rho V \frac{\partial u}{\partial \rho} + uV\right)\frac{d\rho}{d\theta} = -\rho h \frac{dV}{d\theta} + \left(\sum \dot{m}_{in} h_{in} - \sum \dot{m}_{out} h_{out} + \dot{Q}\right)\frac{1}{\omega}$$

h = Specific enthalpy, J/kg
$\dot{m}_{in}$ = Mass flow rate into CV, kg/s
$\dot{m}_{out}$ = Mass flow rate out of CV, kg/s
$\dot{Q}$ = Heat transfer rate into CV, W
T = Temperature, K

u = Specific internal energy, J/kg
V = Volume of chamber, m^3
$\dot{W}$ = Work done by control volume, W
ϑ = Crankshaft angle, degrees
ω = Rotational speed of crank, deg/s
ρ = Density, kg/m^3

Thermodynamic Models:
Conservation of Mass and Energy

- Combined mass and energy balance can be solved in series for $d\rho/d\theta$ and $dT/d\theta$:

(properties) (mass flow) (geometry) (heat transfer)

$$\frac{d\rho}{d\theta} = \frac{1}{V}\left[-\rho\frac{dV}{d\theta} + \frac{1}{\omega}\left(\sum \dot{m}_{in} - \sum \dot{m}_{out}\right)\right]$$

$$\frac{dT}{d\theta} = \frac{-\rho h\frac{dV}{d\theta} - \left(uV + \rho V\frac{\partial u}{\partial \rho}\right)\frac{\partial \rho}{\partial \theta} + \frac{1}{\omega}\left(\dot{Q} + \sum \dot{m}_{in}h_{in} - \sum \dot{m}_{out}h_{out}\right)}{\rho V\frac{\partial u}{\partial T}}$$

Thermodynamic Models:
Conservation of Mass and Energy

- To solve the mass and energy balances, first need sub-models for:
  - Geometry
  - Mass flow
  - Heat transfer
  - Properties
- Then numerically solve mass and energy balance for density and temperature variation with crank angle
  - Use equations of state to solve for any other desired properties

Thermodynamic Models:

Geometry Sub-Model

- Need expressions for volume to solve mass and energy balance:

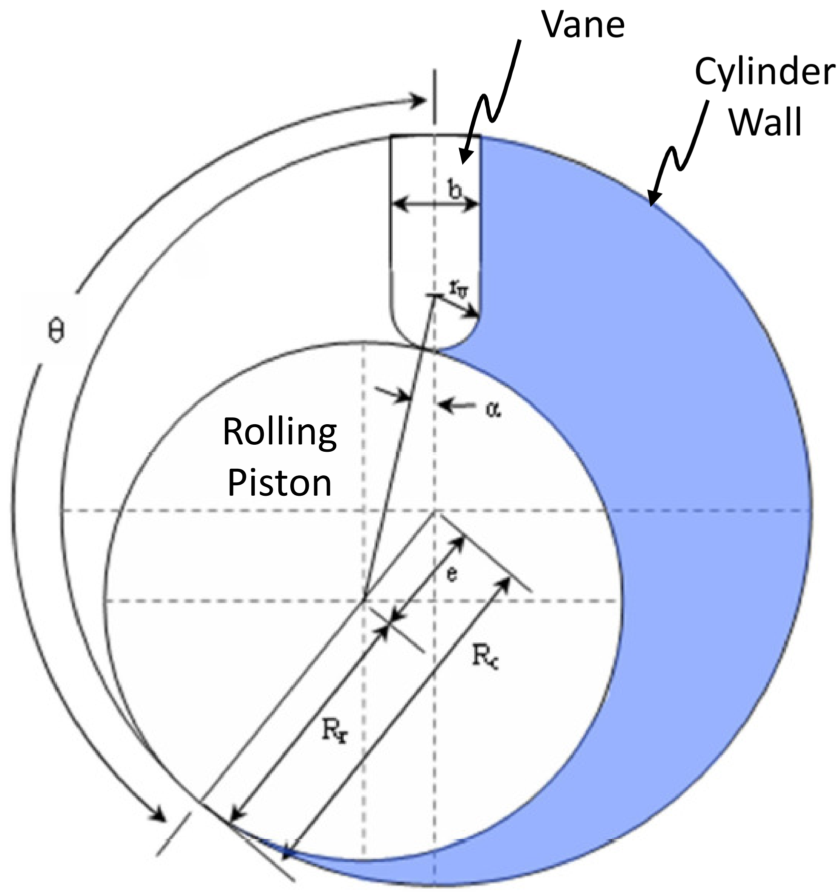
$$\frac{d\rho}{d\theta} = \frac{1}{V}\left[-\rho\frac{dV}{d\theta} + \frac{1}{\omega}\left(\sum \dot{m}_{in} - \sum \dot{m}_{out}\right)\right]$$

$$\frac{dT}{d\theta} = \frac{-\rho h\frac{dV}{d\theta} - \left(uV + \rho V\frac{\partial u}{\partial \rho}\right)\frac{\partial \rho}{\partial \theta} + \frac{1}{\omega}\left(\dot{Q} + \sum \dot{m}_{in}h_{in} - \sum \dot{m}_{out}h_{out}\right)}{\rho V\frac{\partial u}{\partial T}}$$

- Form of volume expression depends on type of compressor analyzed

Thermodynamic Models:
Geometry Sub-Model

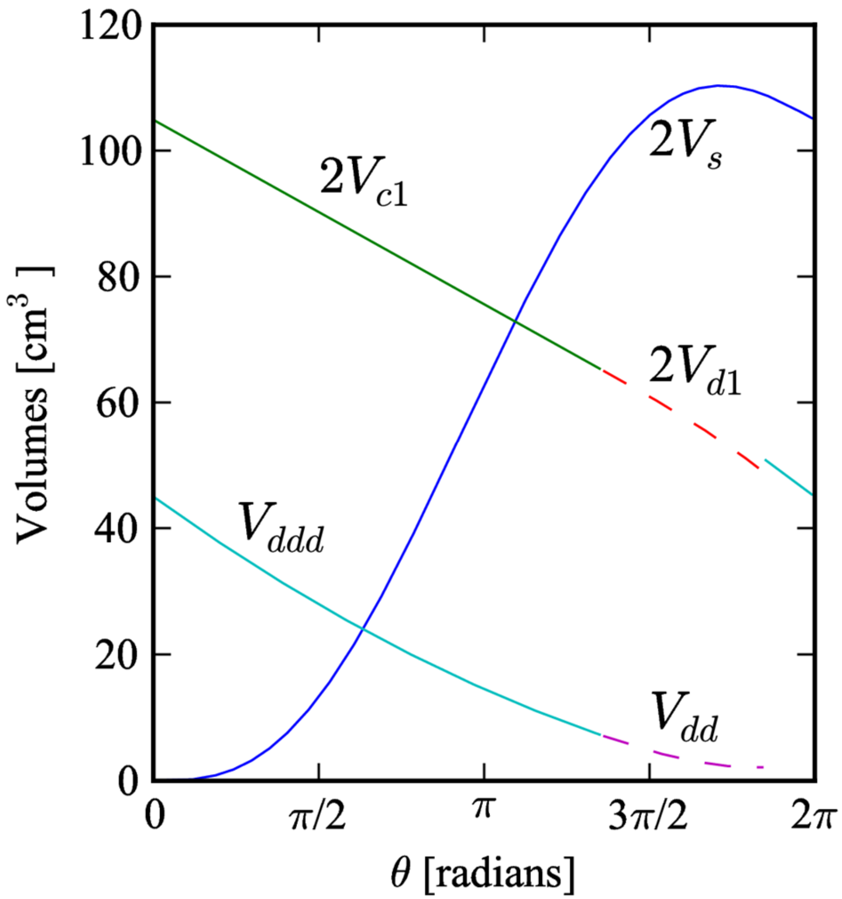
- Expression for volume as a function of crank angle can be derived based on geometry
  - For example, rolling piston volume can be expressed in terms of the geometric parameters shown



Mathison (2011)

Thermodynamic Models:
Geometry Sub-Model

- For more complicated geometries, analysis is non-trivial
- However, many solutions are available in literature
  - For example, Bell et al. (2010) presents scroll analysis



Bell et al. (2010)

Thermodynamic Models:
Mass Flow Sub-Model

- Need expressions for mass flow to solve mass and energy balance:

$$\frac{d\rho}{d\theta} = \frac{1}{V}\left[-\rho\frac{dV}{d\theta} + \frac{1}{\omega}\left(\sum \dot{m}_{in} - \sum \dot{m}_{out}\right)\right]$$

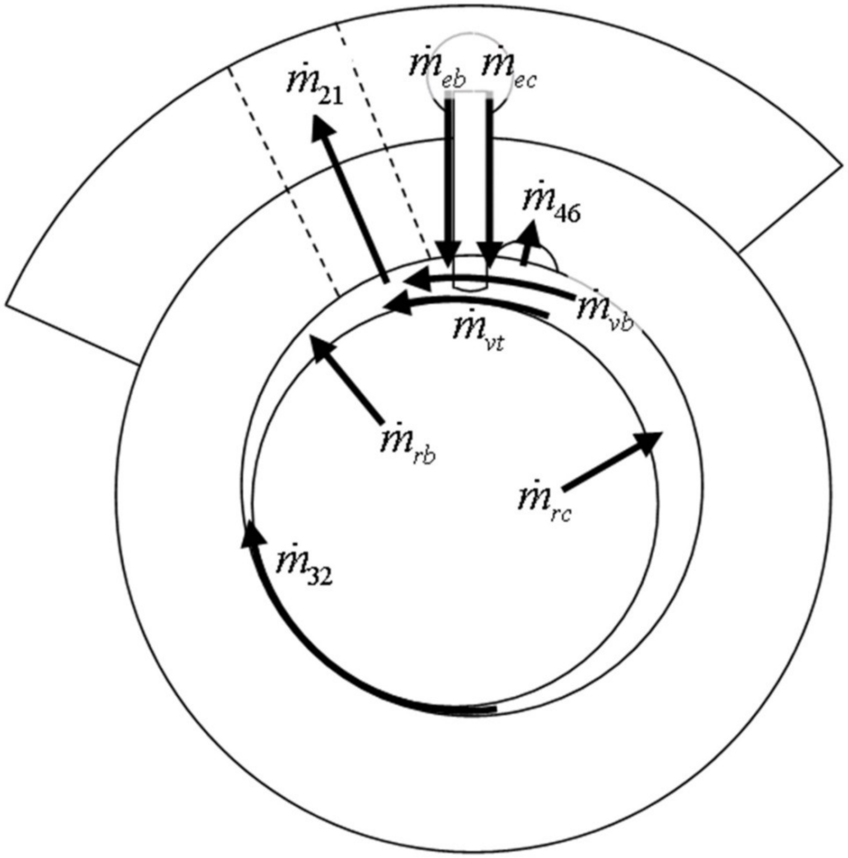
$$\frac{dT}{d\theta} = \frac{-\rho h\frac{dV}{d\theta} - \left(uV + \rho V\frac{\partial u}{\partial \rho}\right)\frac{\partial \rho}{\partial \theta} + \frac{1}{\omega}\left(\dot{Q} + \sum \dot{m}_{in} h_{in} - \sum \dot{m}_{out} h_{out}\right)}{\rho V\frac{\partial u}{\partial T}}$$

- Form of mass flow expression depends on type of flow path

Thermodynamic Models:
Mass Flow Sub-Model

- Mass flow models must estimate suction and discharge flow as well as leakage
  - Driven by pressure differences between CVs
  - Dependent on flow area, which can be a function of crank angle

$$\dot{m} = f\left[P_{high}, P_{low}, \rho_{high}, A(\theta)\right]$$



Mathison (2011)

Thermodynamic Models:
Mass Flow Sub-Model

- Flow models available in literature
  - Isentropic nozzle flow
  - Frictionally corrected isentropic nozzle flow
  - Couette and Poiseuille flow
  - Laminar viscous flow
  - Fanno/Rayleigh flows
  - Empirical relations
- Selection depends on type of flow path, desired accuracy, and available information

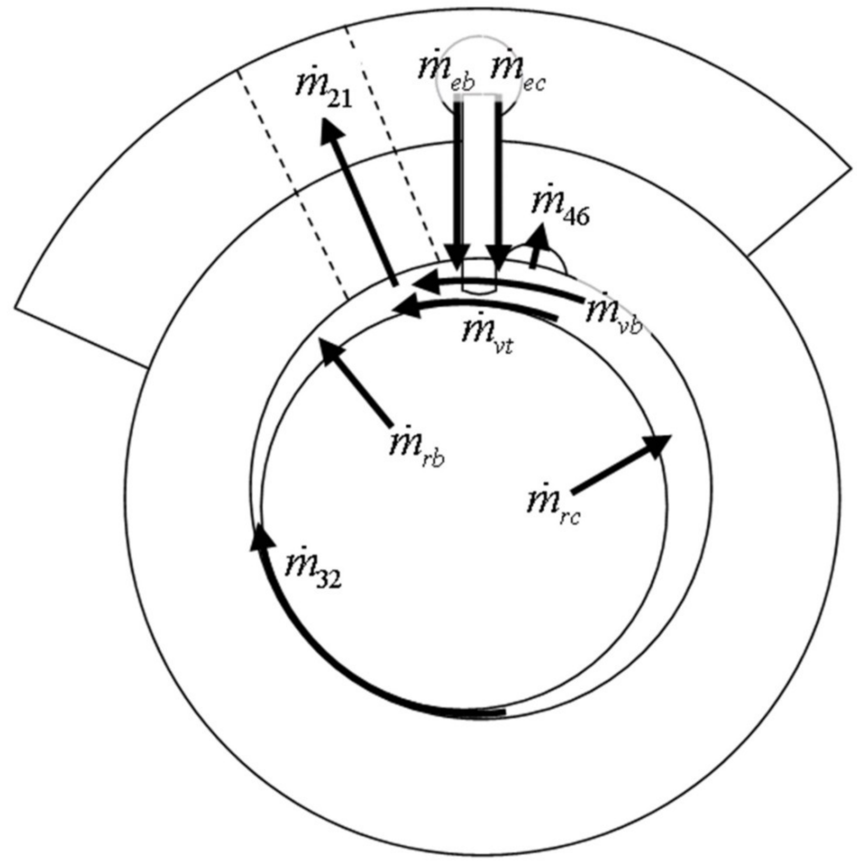
$$\dot{m} = \rho A V$$

$$\frac{p}{p_o} = \left[1 + \frac{\gamma - 1}{2}\left(\frac{V}{c}\right)^2\right]^{-\frac{\gamma}{\gamma - 1}}$$

A =	Flow path area
c =	Speed of sound, $\sqrt{\gamma R T}$
p =	Pressure
p_o =	Back pressure
R =	Gas constant
V =	Flow velocity
γ =	Specific heat ratio
ρ =	Density

Thermodynamic Models:
Valve Sub-Model

- In many compressors, area available for discharge flow depends on valve motion
- If pressure difference drives valve motion, then valve sub-model is required
  - Valve motion typically modeled using classical mechanics

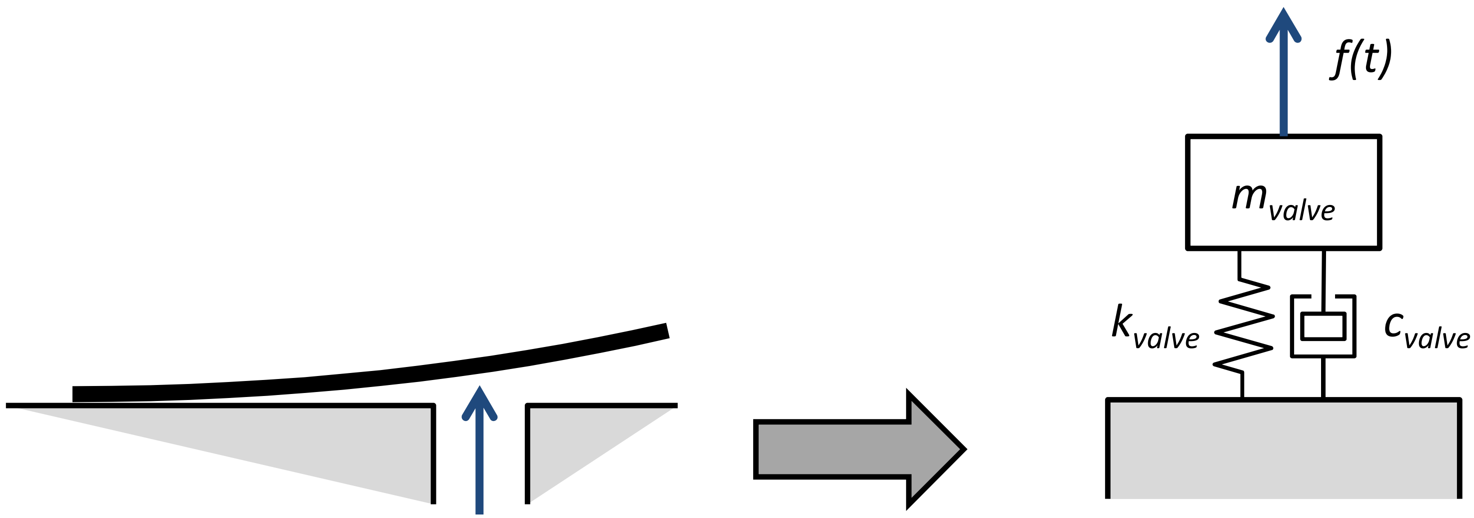


Mathison (2011)

Thermodynamic Models:
Valve Sub-Model

Example: Reed valves modeled as thin, Euler beams

- Single degree-of-freedom lumped element system
- Described by second-order ordinary differential equation



$$m_{valve}\ddot{x}(t) + c_{valve}\dot{x}(t) + k_{valve}x(t) = f(t)$$

Thermodynamic Models:
Heat Transfer Sub-Model

- Need expressions for heat transfer to solve mass and energy balance:

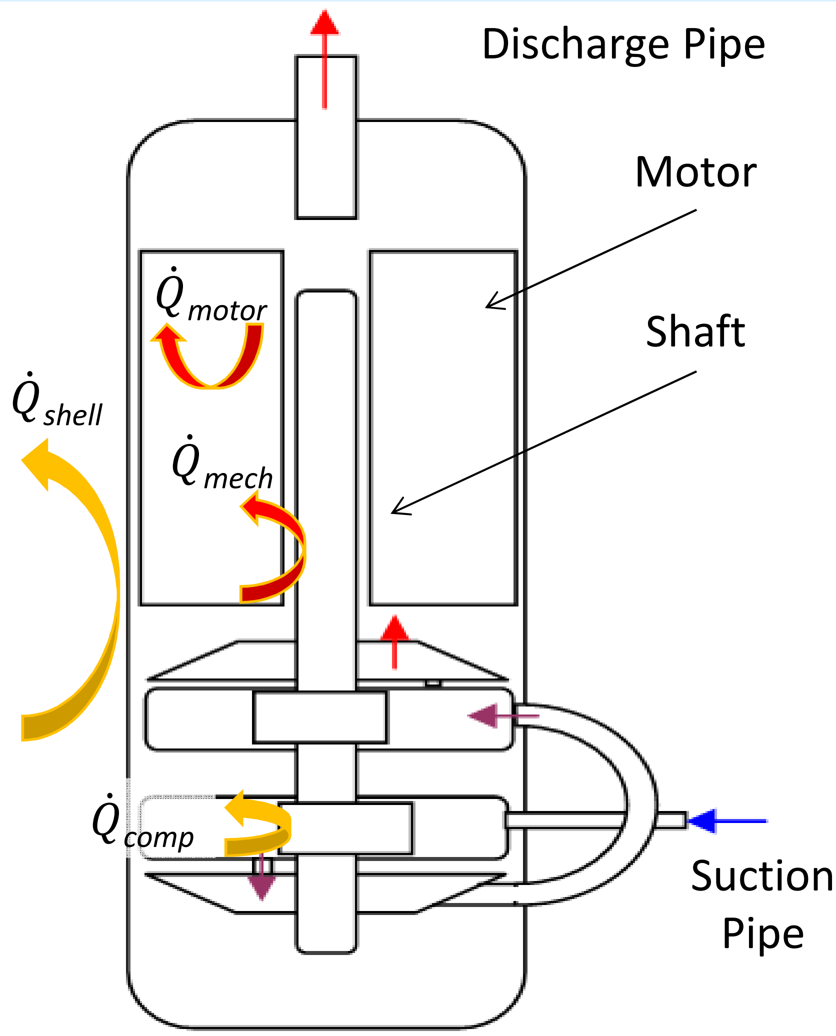
$$\frac{d\rho}{d\theta} = \frac{1}{V}\left[-\rho\frac{dV}{d\theta} + \frac{1}{\omega}\left(\sum \dot{m}_{in} - \sum \dot{m}_{out}\right)\right]$$

$$\frac{dT}{d\theta} = \frac{-\rho h\frac{dV}{d\theta} - \left(uV + \rho V\frac{\partial u}{\partial \rho}\right)\frac{\partial \rho}{\partial \theta} + \frac{1}{\omega}\left(\dot{Q} + \sum \dot{m}_{in}h_{in} - \sum \dot{m}_{out}h_{out}\right)}{\rho V\frac{\partial u}{\partial T}}$$

- Form of heat transfer expression depends on type of heat transfer path

Thermodynamic Models:
Heat Transfer Sub-Model

- In general, the most significant heat transfer paths to the refrigerant involve convection
  - Between refrigerant and interior compressor walls, $\dot{Q}_{comp}$
  - Between shell and surroundings, $\dot{Q}_{shell}$
- May also consider heat dissipated by motor and mechanical losses, $\dot{Q}_{motor}$ and $\dot{Q}_{mech}$



Thermodynamic Models:
Heat Transfer Sub-Model

- Convective heat transfer is modeled by Newton's law of cooling:

$$\dot{Q} = h_c A\left(T_w - T_g\right)$$

where positive $\dot{Q}$ indicates heat transfer to the gas.

A = Surface area, m^2

h_c = Heat transfer coefficient, W/m^2-K

T_g = Temperature of refrigerant, K

T_w = Temperature of wall, K

$\dot{Q}$ = Heat transfer rate, W

Thermodynamic Models:
Heat Transfer Sub-Model

- The exact correlation for heat transfer coefficient depends on the compressor type

- Solution process:
  - Use properties and geometry model to calculate heat transfer coefficient
  - Use wall temperature calculated from an overall, lumped energy balance (or an initial guess for wall temperature) to calculate the instantaneous heat transfer rate:

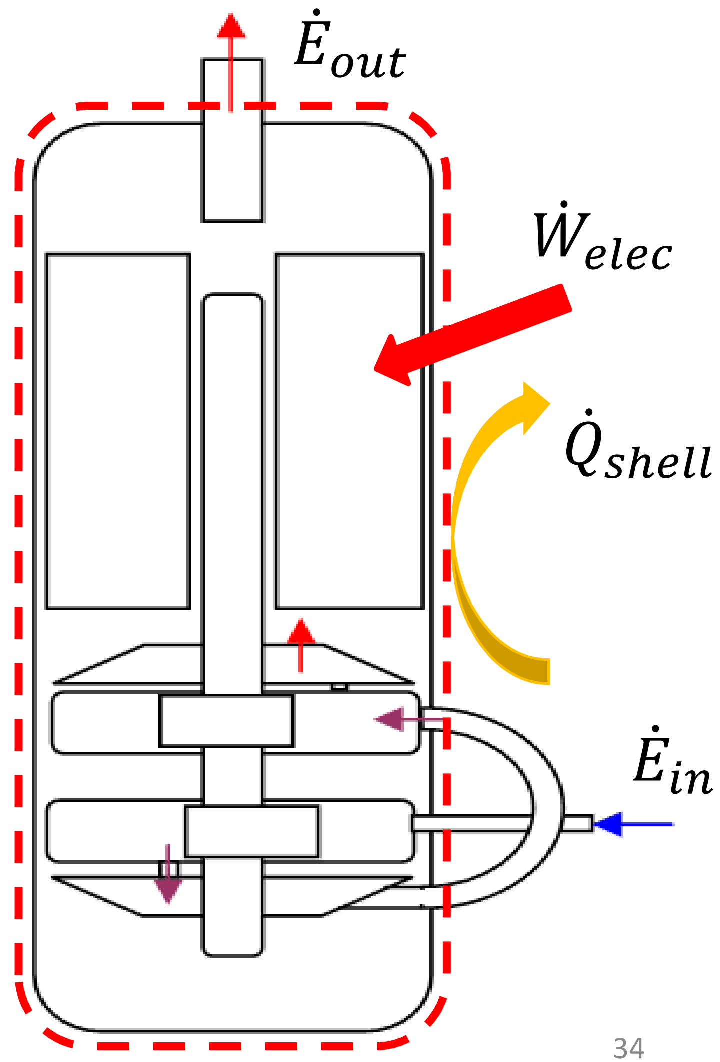
$$\dot{Q} = h_c A\left(T_w - T_g\right)$$

Thermodynamic Models:
Heat Transfer Sub-Model

- Solve overall energy balance for compressor at steady state:

$$\dot{E}_{in} + \dot{W}_{elec} = \dot{E}_{out} + \dot{Q}_{shell}$$

- Iteratively adjust the lumped temperature until the energy balance converges



Thermodynamic Models:
Conservation of Mass and Energy

- With sub-models in place, mass and energy balance can be solved in series for $d\rho/d\theta$ and $dT/d\theta$:

(properties) (mass flow) (geometry) (heat transfer)

$$\frac{d\rho}{d\theta} = \frac{1}{V}\left[-\rho\frac{dV}{d\theta} + \frac{1}{\omega}\left(\sum \dot{m}_{in} - \sum \dot{m}_{out}\right)\right]$$

$$\frac{dT}{d\theta} = \frac{-\rho h\frac{dV}{d\theta} - \left(uV + \rho V\frac{\partial u}{\partial \rho}\right)\frac{\partial \rho}{\partial \theta} + \frac{1}{\omega}\left(\dot{Q} + \sum \dot{m}_{in}h_{in} - \sum \dot{m}_{out}h_{out}\right)}{\rho V\frac{\partial u}{\partial T}}$$

Thermodynamic Models:
Numerical Solution

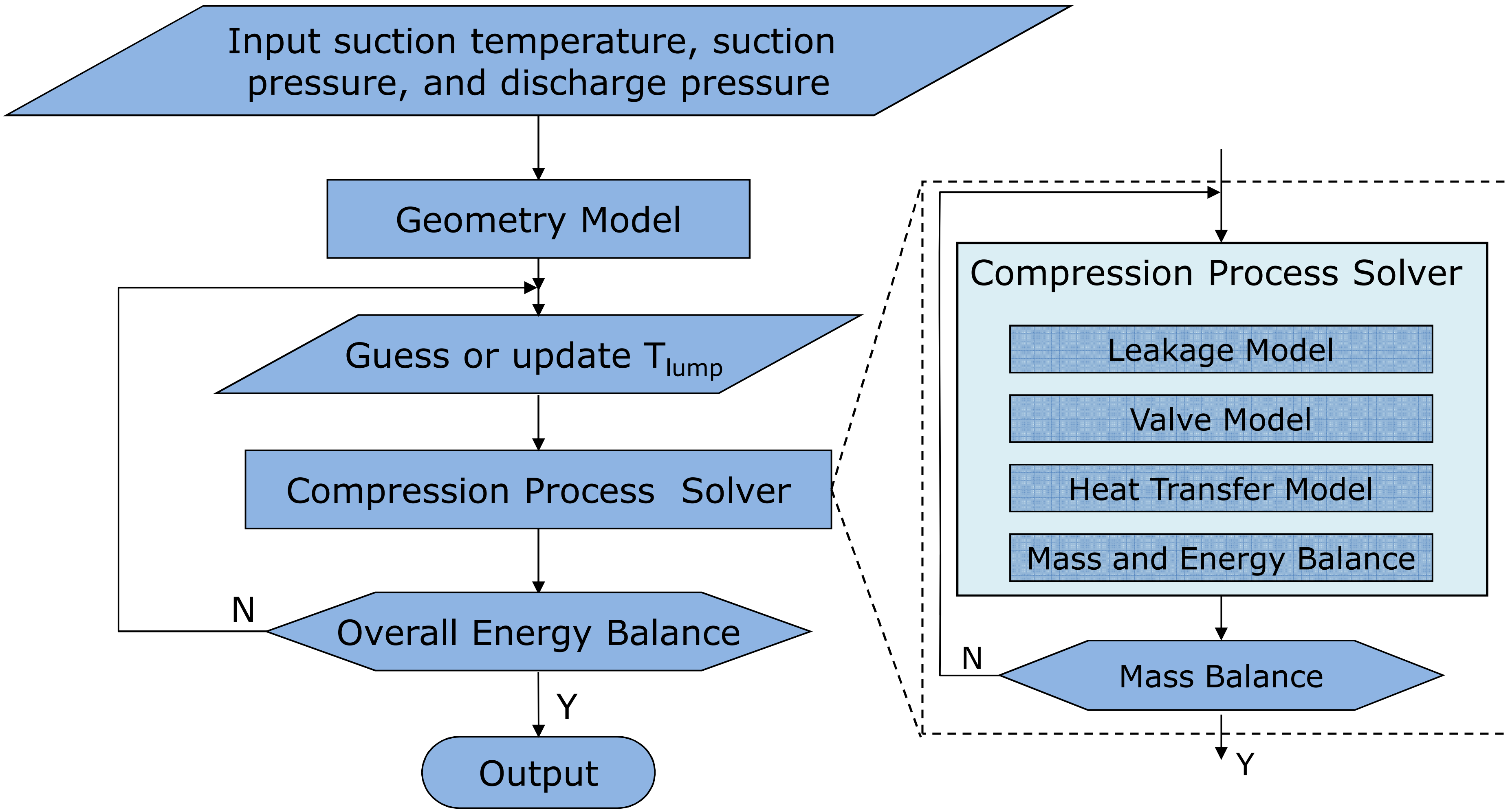
- Derivatives can be numerically integrated to determine properties at the next time step
  - For example, using the Euler method,

$$\rho(\theta + \Delta\theta) = \rho(\theta) + \Delta\theta \cdot \left.\frac{d\rho}{d\theta}\right|_{\theta}$$

$$T(\theta + \Delta\theta) = T(\theta) + \Delta\theta \cdot \left.\frac{dT}{d\theta}\right|_{\theta}$$

  - All other properties can be evaluated based on ρ and T
- Requires a guess for the conditions in each CV at the beginning of the crank shaft rotation
- Solution process must be repeated until conditions at the beginning and end of the rotation converge

Thermodynamic Models:
Numerical Solution



Thermodynamic Models:
Validation

- Models require experimental validation
  - Data can be used to tune the model for improved accuracy
- Validated model can be used for parametric studies
  - Trends can be predicted due to physical basis of model
- Should be able to predict
  - Power and mass flow rate within ±5%
  - Discharge temperature within ±10℃

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- Motivation
- Types of compressor models
- Components of a thermodynamic model
- **Incorporating alternative refrigerants**
- Case study
  - R-410A, R-32, ammonia, and CO_2
- Conclusions

Alternative Refrigerants

- The compression process equations require property data for the working fluid:

$$\frac{d\rho}{d\theta} = \frac{1}{V}\left[-\rho\frac{dV}{d\theta} + \frac{1}{\omega}\left(\sum \dot{m}_{in} - \sum \dot{m}_{out}\right)\right]$$

$$\frac{dT}{d\theta} = \frac{-\rho h\frac{dV}{d\theta} - \left(uV + \rho V\frac{\partial u}{\partial \rho}\right)\frac{\partial \rho}{\partial \theta} + \frac{1}{\omega}\left(\dot{Q} + \sum \dot{m}_{in}h_{in} - \sum \dot{m}_{out}h_{out}\right)}{\rho V\frac{\partial u}{\partial T}}$$

- The source of property data depends on the type of working fluid

Alternative Refrigerants

Sources of Thermophysical Property Data:

- Equations of state (EOS)
  - Ideal gas EOS
- Property databases
  - REFPROP
  - CoolProp
- Empirical equations fitted to available data

Additional Considerations:

- Changes in lubricant properties or concentration impact leakage between control volumes

Alternative Refrigerants

Additional Considerations:

- Changes in lubricant properties or concentration impact conservation of mass and energy if the equations are modified to account for oil:

$$\frac{dV_r}{d\theta} = \frac{dV}{d\theta} - \frac{dV_o}{d\theta}$$

where properties without a subscript are for the CV as a whole,
properties with the subscript "r" are for the refrigerant,
properties with the subscript "o" are for the oil.

Alternative Refrigerants

Energy balance without oil:

$$\frac{dT}{d\theta} = \frac{-\rho h \frac{dV}{d\theta} - \left(uV + \rho V \frac{\partial u}{\partial \rho}\right) \frac{\partial \rho}{\partial \theta} + \frac{1}{\omega}\left(\dot{Q} + \sum \dot{m}_{in} h_{in} - \sum \dot{m}_{out} h_{out}\right)}{\rho V \frac{\partial u}{\partial T}}$$

Energy balance with oil:

$$\frac{dT}{d\theta} = \frac{\boxed{\left[-cT \frac{dm}{d\theta} - p \frac{dV}{d\theta}\right]_o} + \left[-\rho h \frac{dV}{d\theta} - \left(uV + \rho V \frac{\partial u}{\partial \rho}\right) \frac{\partial \rho}{\partial \theta}\right]_r + \frac{1}{\omega}\left(\dot{Q} + \sum \dot{m}_{in} h_{in} - \sum \dot{m}_{out} h_{out}\right)}{\boxed{[\rho V c]_o} + \left[\rho V \frac{\partial u}{\partial T}\right]_r}$$

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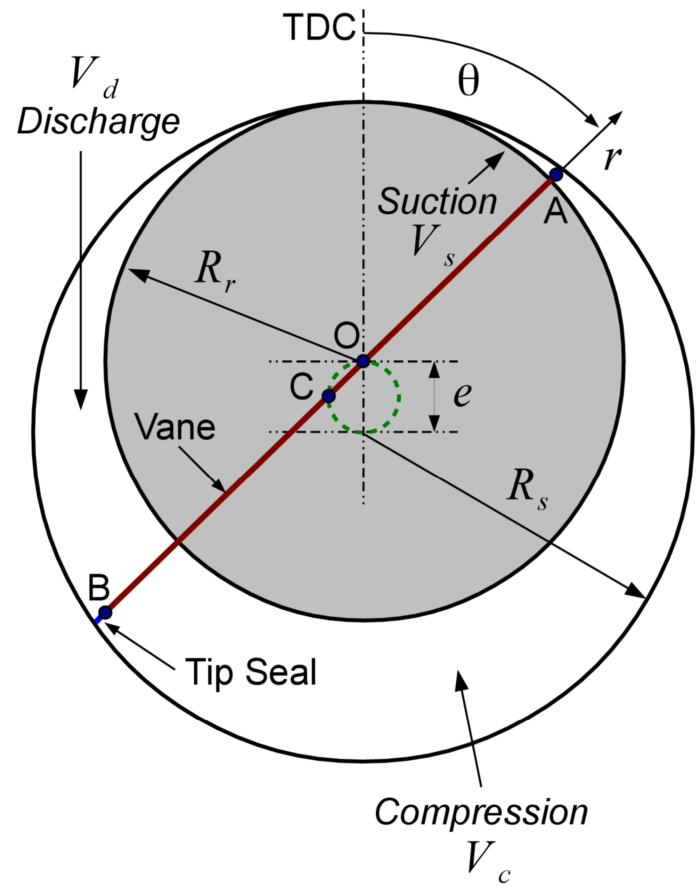
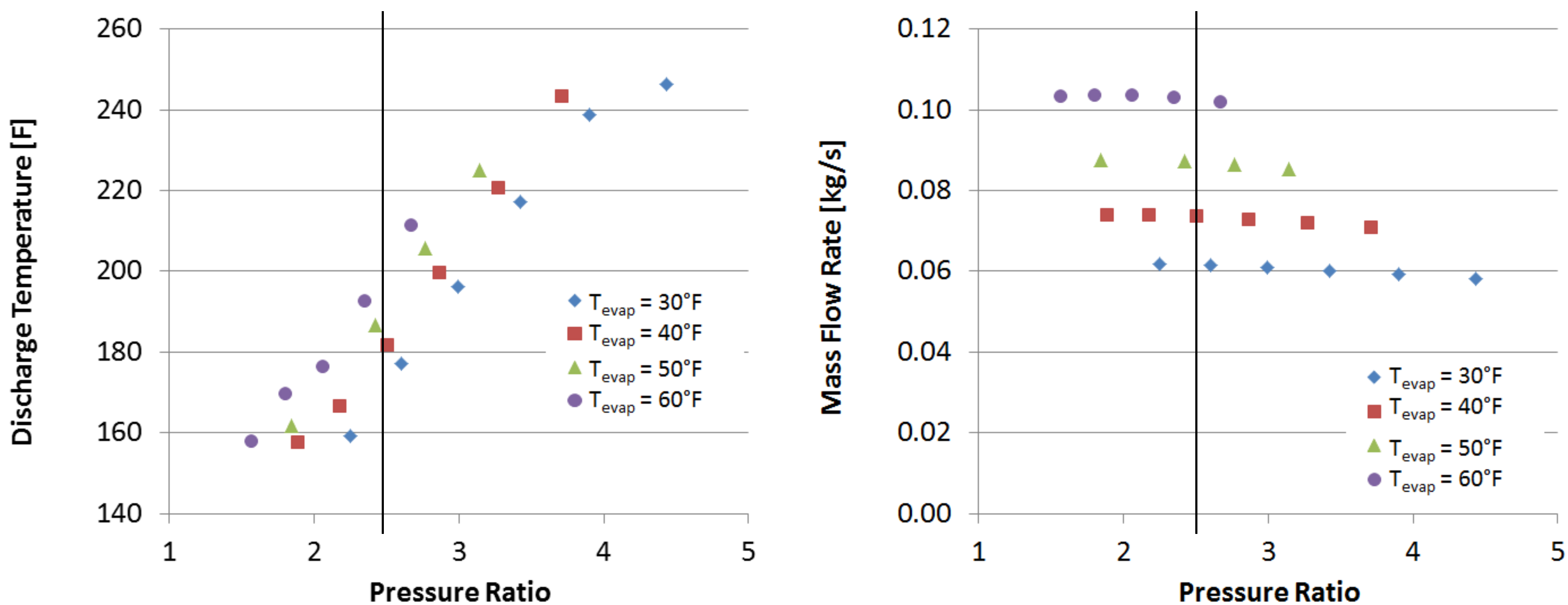
Case Study:
Spool Compressor

- Rotary vane type compressor with rotating faceplates
- Model includes physically based sub-models of:
  - Friction losses
  - Semi-hermetic, high pressure shell
- Model uses simplified sub-model for valve that neglects dynamics



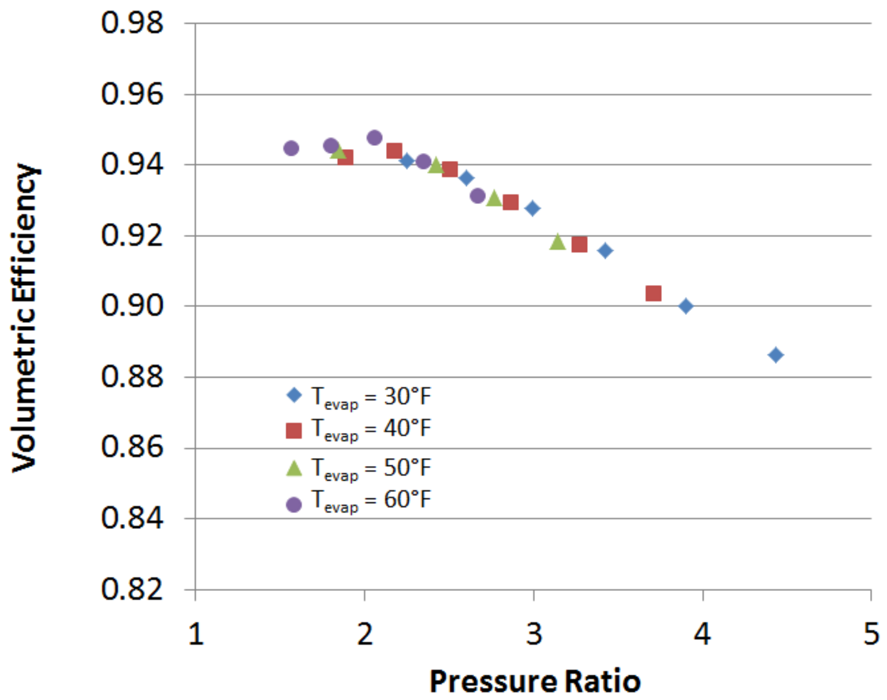
Bradshaw and Groll (2013)

Case Study:
Spool Compressor with R-32



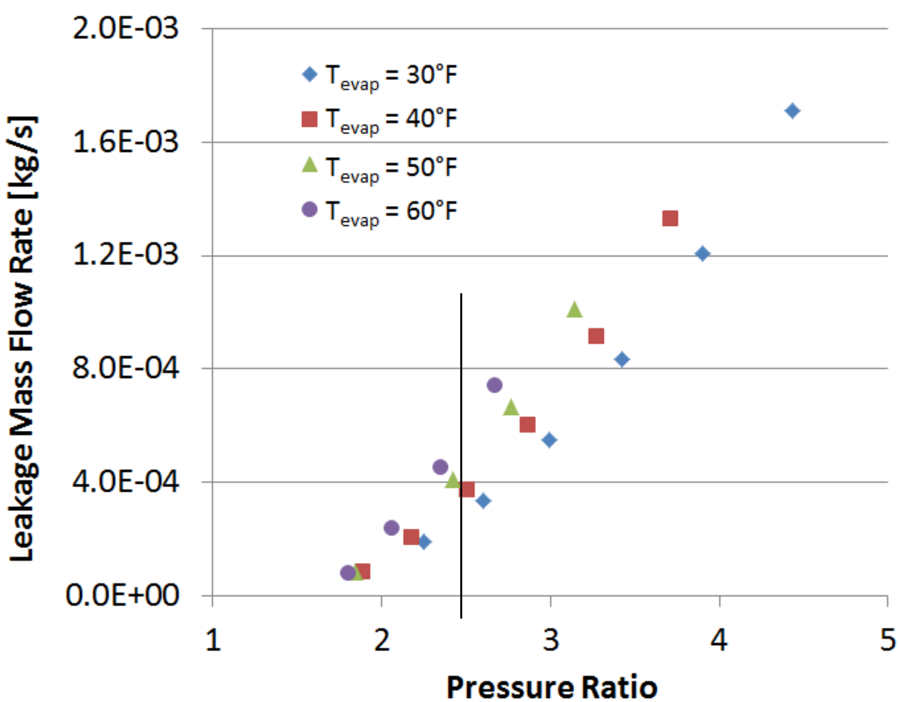
Model can predict trends in discharge temperature and mass flow rate over a range of operating conditions.

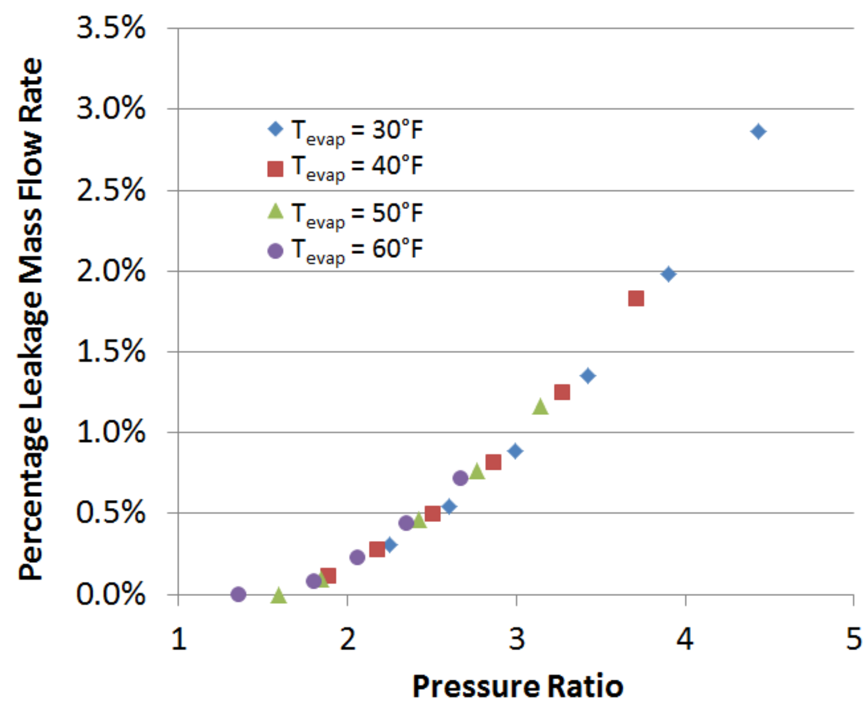
Case Study:
Spool Compressor with R-32



Model can predict trends in volumetric efficiency over a range of operating conditions.

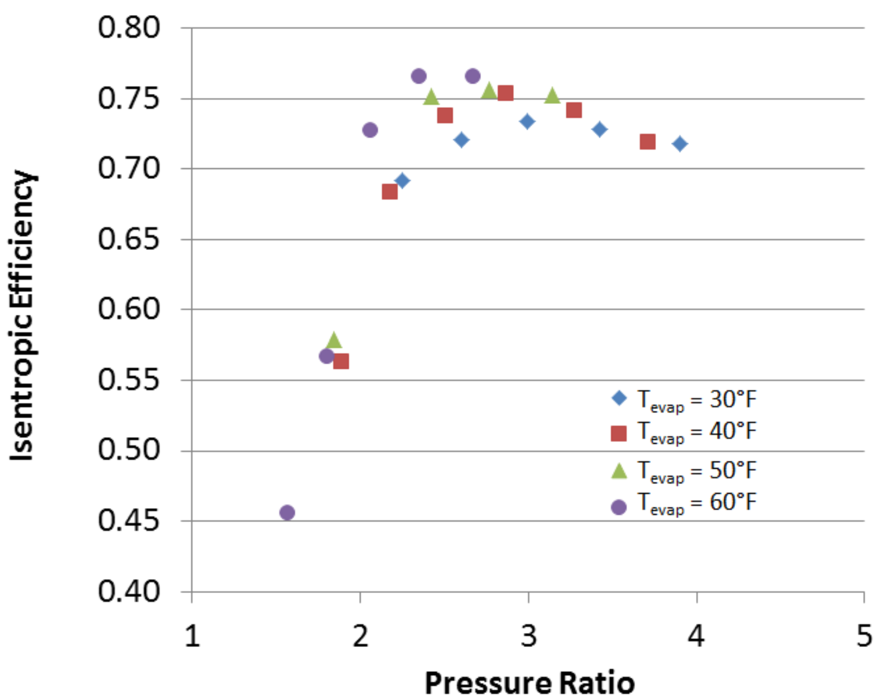
Case Study:
Spool Compressor with R-32





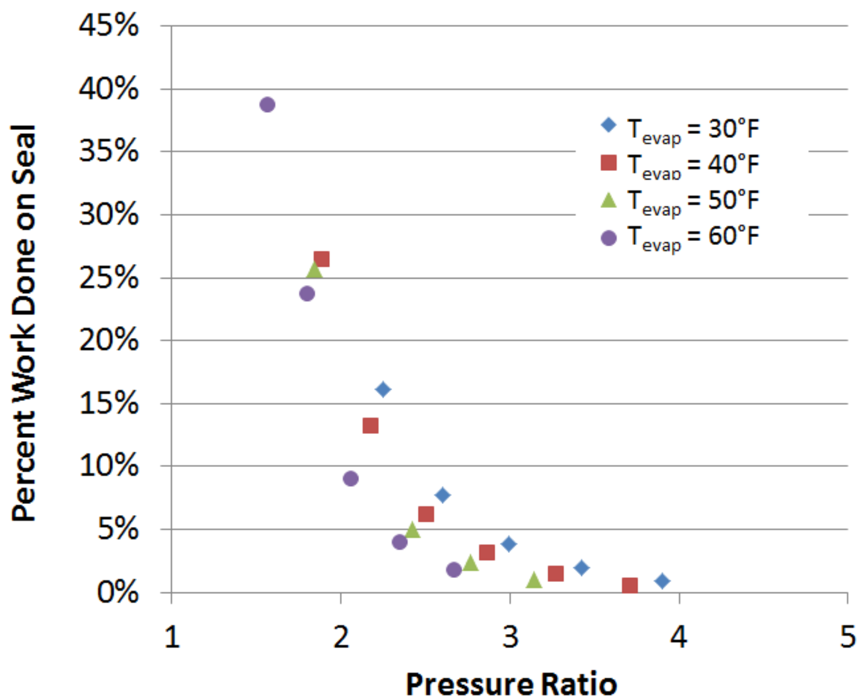
Model reveals trends in seal leakage flow rates that impact volumetric efficiency

Case Study:
Spool Compressor with R-32



Model can predict optimal pressure ratio for maximizing isentropic efficiency

Case Study:
Spool Compressor with R-32

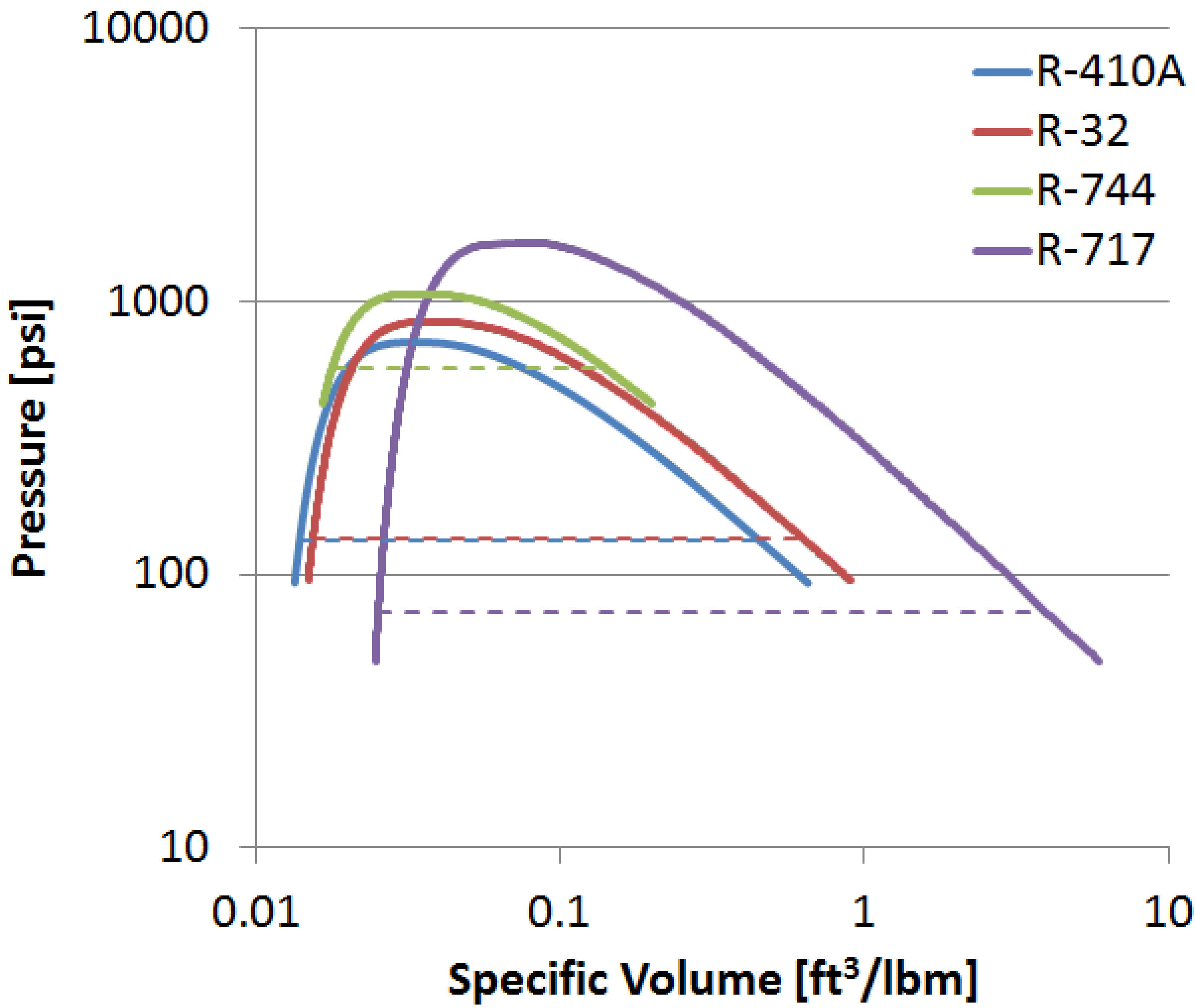


Model reveals trends in seal friction that impact isentropic efficiency

Case Study:
Comparison of Refrigerants

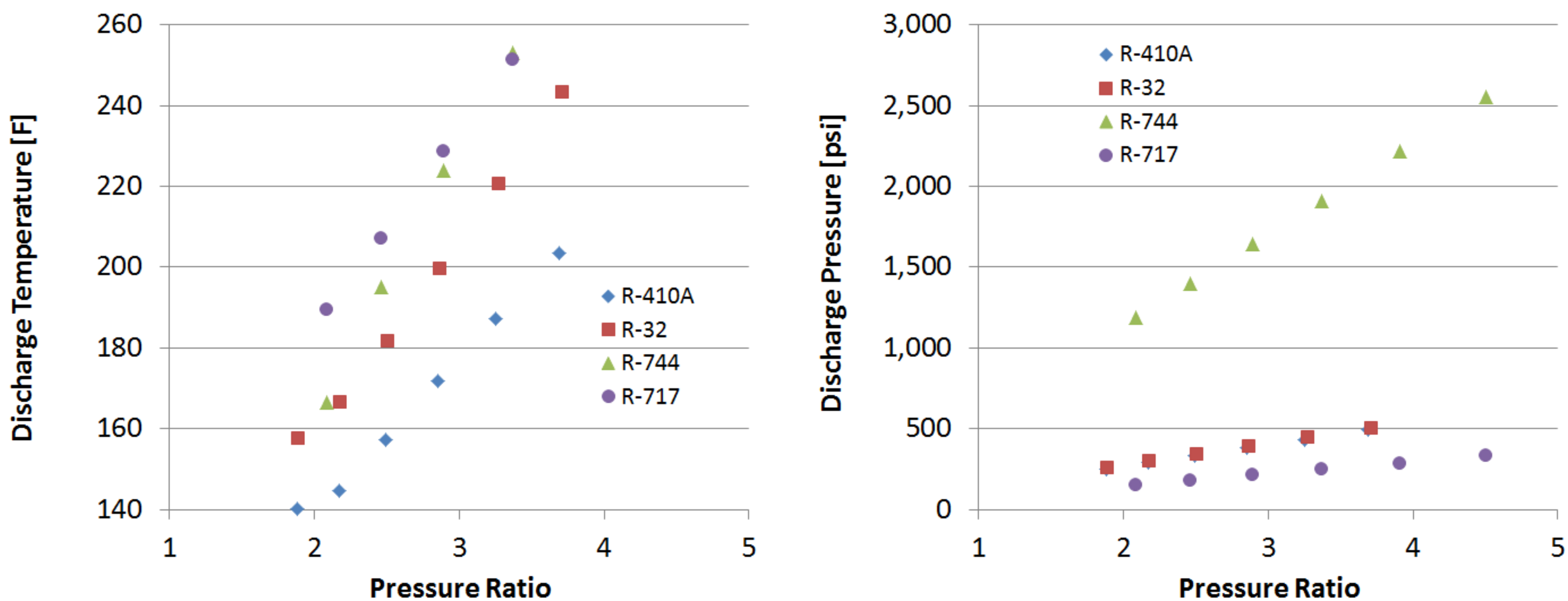
- Compressor performance is compared for four refrigerants assuming:
  - No changes in compressor geometry
  - No changes in oil properties

Case Study:
Comparison of Refrigerants



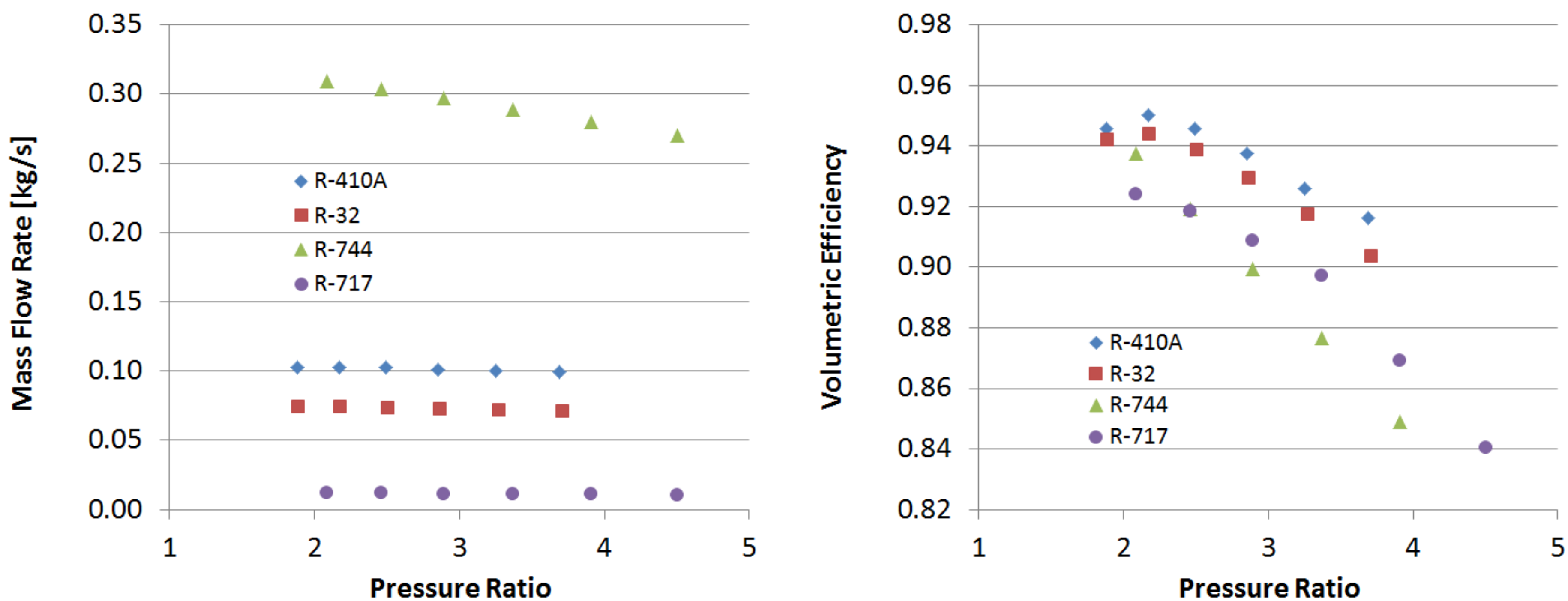
Dashed lines show saturated liquid-vapor mixture properties at 40°F.

Case Study:
Comparison of Refrigerants



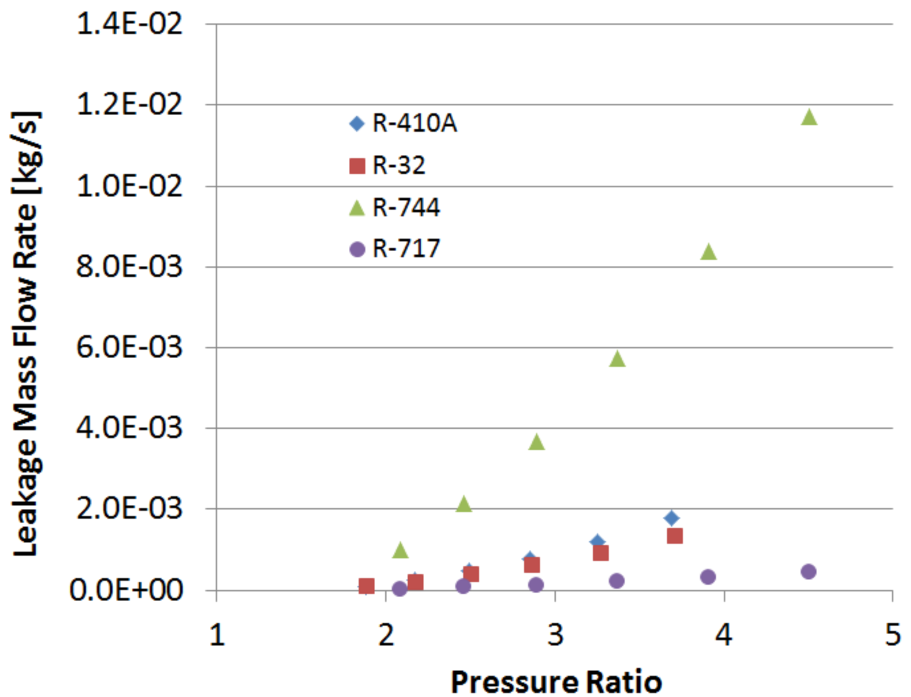
Impact of working fluid on discharge conditions while holding evaporating temperature constant at 40℉.

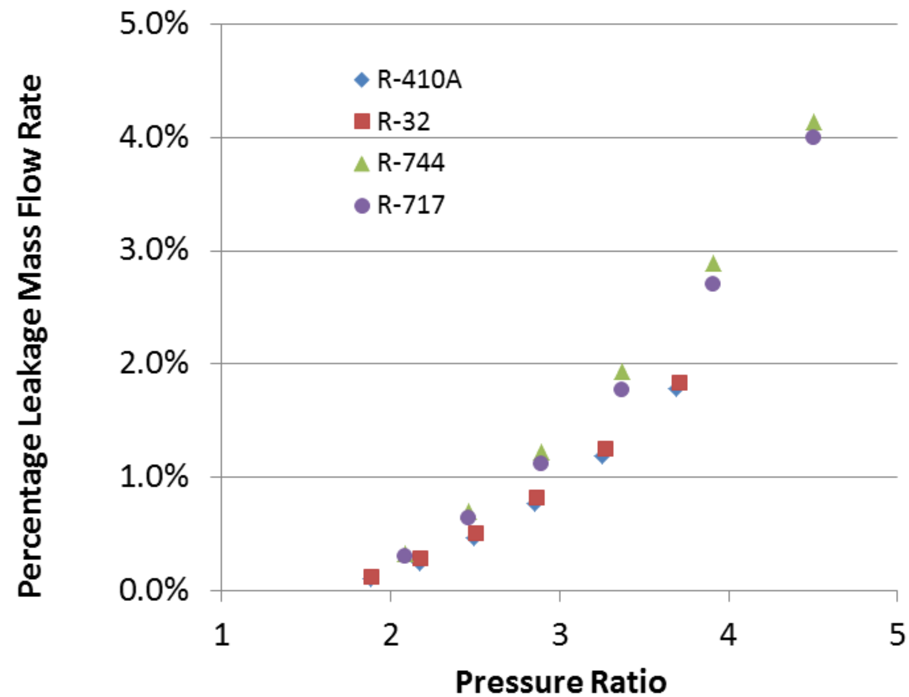
Case Study:
Comparison of Refrigerants



Impact of working fluid on mass flow rate and volumetric efficiency while holding evaporating temperature at 40℉.

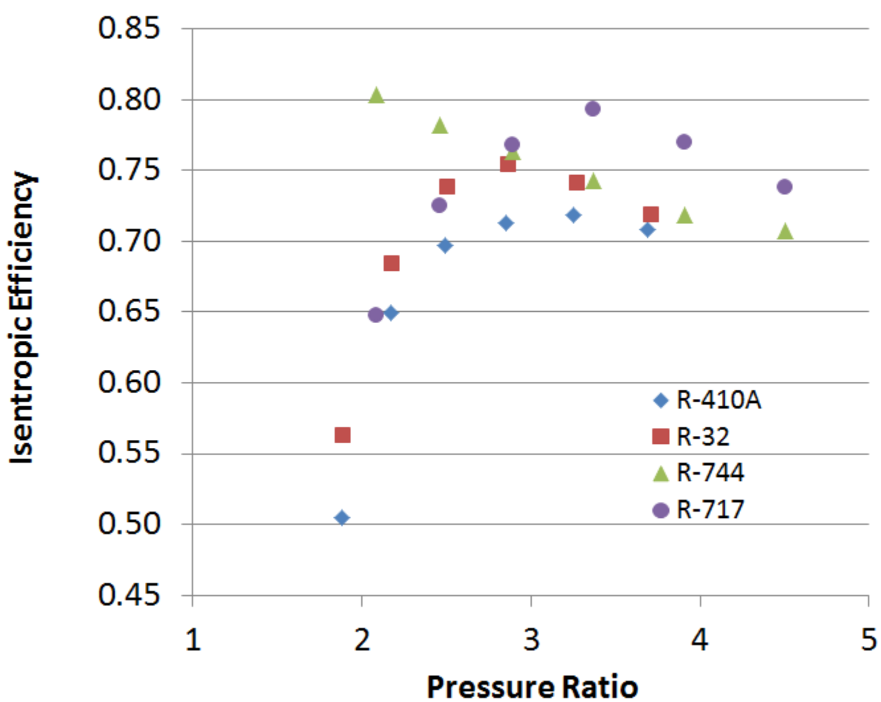
Case Study:
Comparison of Refrigerants

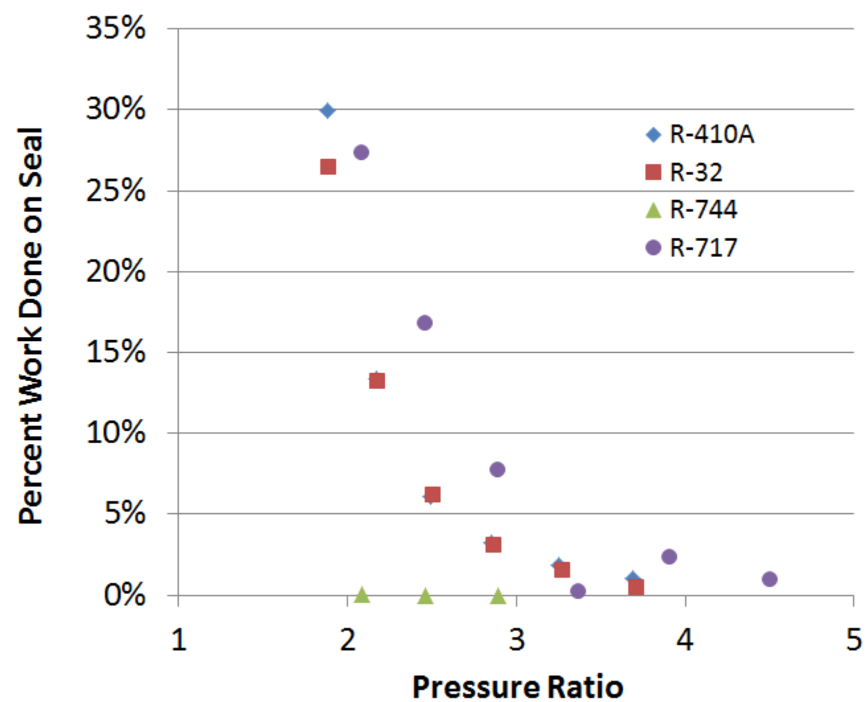




Impact of working fluid on face seal leakage flow rate while holding evaporating temperature constant at 40℉.

Case Study:
Comparison of Refrigerants





Impact of working fluid on isentropic efficiency and seal friction while holding evaporating temperature at 40℉.

Conclusions

Thermodynamic Compressor Models

- Provide a time- and cost-effective method for evaluating impact of alternative refrigerants
  - Modifying working fluid requires very little effort
- Require experimental validation
- Can predict performance trends due to use of physically-based models
- Achieve sufficient accuracy for parametric studies
  - Power and mass flow rate within ±5%
  - Discharge temperature within ±10℃

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Questions?

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